## OPTIMUM CONDITIONS FOR FORMATION OF SHORT AND LONG RADIATION PULSES IN CO<sub>2</sub> LASERS

## V. N. Tishchenko

UDC 621.378.334

At present CO<sub>2</sub> lasers excited by electric discharges have obtained considerable development. High values of the efficiency and the specific and absolute radiation energy have been achieved [1, 2]. Theoretical methods of investigation of CO<sub>2</sub> lasers are being developed intensively, making it possible to determine the dynamics of excitation and generation and the effect of the composition on the main parameters of lasers [1, 3-8]. In doing this considerable attention is paid to questions of the optimization of the mode of excitation of laser systems possessing a certain general set of essential distinguishing features. A typical example of such a separate class of laser systems is comprised of pulsed electric-discharge CO<sub>2</sub> lasers operating with a high content of CO<sub>2</sub> in the mixture. In this case one can obtain radiation pulses with a duration of ~  $10^{-7}$  sec at a considerable radiation energy density, which allows their use in experiments on plasma heating [9].

The purpose of the present report is to study the optimum conditions for the formation of short and long radiation pulses in  $CO_2$  lasers of the indicated type. As the characteristic scale for the characterization of the duration of the radiation it is convenient to choose the relaxation time of the upper laser level  $\tau \simeq 5 \cdot 10^{-6}$  sec  $\cdot$  atm. In this case the short and long radiation pulses will be determined by the relationships  $pt_r \ll \tau$  and  $pt_r \gg \tau$ , respectively, where the values of  $pt_r \simeq 10^{-7}$  and  $3 \cdot 10^{-5}$  sec  $\cdot$  atm were specifically chosen in the calculations.

The search for the optimum modes of transformation of electrical energy into coherent radiation was carried out with the help of mathematical modeling of the laser processes. A system of equations describing pumping of the active medium by electrons of the discharge, induced emission on the P(20) vibrational-rota-tional transition, heating of the gas, and relaxation processes was solved on a computer for the following transitions:

$$\begin{split} \mathrm{N_2(1)} + \mathrm{CO_2(00^00)} \rightleftharpoons \mathrm{CO_2(00^01)} + \mathrm{N_2(0)}, \\ \mathrm{CO_2(00^01)} + M \rightleftharpoons \begin{cases} \mathrm{CO_2(11^{10})} + M \\ \mathrm{CO_2(03^{10})} + M, \\ \mathrm{CO_2(01^{10})} + M \rightleftharpoons \mathrm{CO_2(00^{00})} + M, \\ \mathrm{CO_2(10^{00})} + \mathrm{CO_2} \rightleftharpoons \mathrm{CO_2(02^{00})} + \mathrm{CO_2}, \end{split}$$

where M is any of the molecules  $CO_2$ ,  $N_2$ , He,  $H_2$ , or  $H_2O$ . For simplicity only the transitions between lower vibrational levels are indicated. The model also takes into account single- and multiquantum transitions between higher vibrational states. The relaxation of the vibrational energy was described using the equations proposed in [5]. The model used in the calculations does not allow for expansion of the gas and its thermal conductivity, which is valid for generation durations of  $\leq 50 \ \mu$ sec. A detailed description of the mathematical model is presented in [10]; more exact data for the rate constants of energy relaxation from the asymmetric mode of  $CO_2$  and for the broadening constants of the  $P(20)CO_2(00^01) - CO_2(10^00)$  [11, 12] were used to determine the coefficient of amplification in the present report.

The mathematical solution of the stated problem is considerably simplified by the introduction of the following parameters (which eliminate the pressure of the active medium from the analysis): the ratio  $W/p^2$  of the specific power of the discharge to the square of the initial pressure of  $CO_2 + N_2$ , the product  $t_pp$  of the duration of pumping times the pressure, and the relative electric field strength E/p in the discharge. In [10] it is shown that a change in the pressure of the active medium does not affect the coefficient of amplification, the efficiency, or the quantities  $Q_r/p$ ,  $I/p^2$ , and  $pt_r$  if the parameters  $W/p^2$ ,  $pt_p$ , and E/p (other conditions being equal) do not vary with the transition to another pressure (I and  $Q_r$  are the radiation intensity and the radiation

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 9-15, May-June, 1977. Original article submitted June 3, 1976.

This material is protected by copyright registered in the name of Plenum Publishing Corporation, 227 West 17th Street, New York, N.Y. 10011. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for \$7.50.



Fig. 1



energy removed from a unit volume of active medium). Therefore, the data presented in the report are correct for p = (0.1-5) atm, although the calculations were made with p = 2 atm for a mixture of  $CO_2 + N_2$ . It should be noted that the mode of pumping and generation can be determined not only by the parameters  $W/p^2$  and  $pt_p$ , but also by one of them and by the relative density Q/p of electrical energy absorbed in the discharge, since  $(W/p^2)pt_p = Q/p$ . In a study of  $CO_2$  lasers having a non-self-maintained discharge it is convenient to introduce (in place of  $W/p^2$ ) the parameter n/p, the relative concentration of electrons in the discharge, since  $W/p^2 = A(n/p)(E/p)^2$ , and with a fixed E/p we have  $W/p^2 \sim n/p$  (A is a constant). The parameters Q/p and  $pt_p$  or n/pand  $pt_p$  were used in the present report.

A typical CO<sub>2</sub> pulse generator was studied: The length of the active medium was L=100 cm, the ratio of L to the length of the resonator equalled 0.8, the distributed losses were taken as equal to  $10^{-4}$  cm<sup>-1</sup>, and one resonator mirror was fully reflecting while the other was semitransparent (coefficient of reflection 0.38). It was assumed that the excitation of the active medium takes place with an electric field strength and electron

concentration n in the discharge which are constant in time. The quantity n is set by an external ionization source.

The calculations showed that for the formation of long  $(pt_r = 3 \cdot 10^{-5} \text{ sec} \cdot \text{atm})$  and short  $(pt_r = 10^{-7} \text{ sec} \cdot \text{atm})$  radiation pulses the most suitable prove to be mixtures in which the ratio between CO<sub>2</sub> and N<sub>2</sub> equals 1:2 and 1:1/4, respectively. The effect of the gases He, H<sub>2</sub>, and H<sub>2</sub>O on the generation efficiency in the indicated modes of duration was also studied. It was established that the following ratios are optimal for obtaining a long radiation pulse in the medium  $1\text{CO}_2:2N_2:p_{\text{He}}=0.4p$ ,  $p_{\text{H}_2}=0.04p$ ,  $p_{\text{H}_2\text{O}}=2\cdot10^{-3}p$ . The formation of pulses with a duration  $pt_r = 10^{-7} \text{ sec} \cdot \text{atm}$  requires rapid pumping of the active medium  $(pt_p = 10^{-7} \text{ sec} \cdot \text{atm})$  with Q/p =  $0.5-1 \text{ J/cm}^3 \cdot \text{atm}$ , as will be shown below. In this case the optimum content of auxiliary gases for a composition of  $1\text{CO}_2:1/4N_2$  equals:  $p_{\text{He}}=p$ ,  $p_{\text{H}_2}=0.1p$ ,  $p_{\text{H}_2}O=2\cdot10^{-2}p$ . (in all cases it is assumed that only one of the gases, He, H<sub>2</sub>, or H<sub>2</sub>O, is added to the active medium.) As the calculations showed, the optimum partial pressures of He, H<sub>2</sub>, and H<sub>2</sub>O depend weakly on the relative electric field strength  $E/p = (10-20) \text{ V/cm} \cdot \text{mm}$  Hg and the density of the energy  $Q/p = (0.01-1) \text{ J/cm}^3 \cdot \text{atm}$  absorbed in the discharge.

The efficiencies of the transformation of electrical energy into coherent radiation with the optimum compositions of the active medium are presented in Fig. 1, where the solid lines correspond to a relative electric field strength  $E/p=10 \text{ V/cm} \cdot \text{mm}$  Hg while the dashed lines correspond to  $E/p=20 \text{ V/cm} \cdot \text{mm}$  Hg. Curves 1-4 pertain to the mixture  $1CO_2:1/4N_2:1.25He$  (mode of short pulses). In these calculations  $p_p$  was fixed and the electron concentration in the discharge was varied. In cases 1 and 2  $p_{tp}=1.3 \cdot 10^{-6}$ , while in 3 and 4  $p_{tp}=$  $1.3 \cdot 10^{-7} \sec \cdot \text{atm}$ . Curves 5-7 pertain to the mixture  $1CO_2:2N_2:6 \cdot 10^{-3}H_2O$  (mode of long pulses). In the calculation of these dependences the pumping durations were also fixed ( $p_{tp}=4 \cdot 10^{-5}$  and  $6 \cdot 10^{-5} \sec \cdot \text{atm}$ ) and the electron concentration in the discharge, and hence the relative density of energy absorbed in the discharge (Q/p) was varied. An increase in the electron concentration leads to the fact that at some value of n the generation is cut off at a time  $pt' < p_p$ . The decrease in pt' is due to the fact that an increase in the power of the discharge leads to faster heating of the gas and populating of the lower laser level and consequently to cutting off of generation. Obviously, the active medium should not be excited for pt > pt'. The values of  $p_t_p$  and pt'corresponding to several values of Q/p are indicated by numbers near the dots on curves 5-7 (see Fig. 1). In all cases the value of the efficiency pertains to the time when generation ceases.

As seen from Fig. 1, the highest efficiency of 0.32 is reached in a mixture of  $1CO_2: 1/4N_2: 1.25He$  with  $Q/p=0.4 \text{ J/cm}^3 \cdot \text{atm}$  and  $E/p=10 \text{ V/cm} \cdot \text{mm}$  Hg. With  $E/p=20 \text{ V/cm} \cdot \text{mm}$  Hg the efficiency is considerably lower, which is due to the lower efficiency of the transformation of electrical energy into the vibrational excitation of the asymmetric mode of  $CO_2$  [4]. In a mixture of  $1CO_2: 2N_2: 6\cdot 10^{-6}\text{H}_2\text{O}$  the maximum efficiency is reached with  $Q/p=0.35 \text{ J/cm}^3 \cdot \text{atm}$  and  $E/p=10 \text{ V/cm} \cdot \text{mm}$  Hg. The effect of the electric field strength on the efficiency and the density of energy removed from a unit volume in this mixture in a quasisteady mode of generation can be judged from Fig. 2, where dependences of the efficiency,  $Q_r/p$ ,  $pt_p$ , and Q/p on the relative electron concentration n/p in the discharge are presented. The solid lines correspond to  $E/p=10 \text{ V/cm} \cdot \text{mm}$  Hg and the dashed lines to  $E/p=20 \text{ V/cm} \cdot \text{mm}$  Hg. In this case it is assumed that pumping of the active medium takes place up to the moment of cutoff of generation, and therefore the dependence of  $pt_p$  on n/p allows one to determine the pumping modes in which one reaches the maximum value of the radiant energy removed from a unit volume of active medium.

As seen from Fig. 2, in the region of  $n/p \le 10^{13} \text{ cm}^{-3} \cdot \text{atm}^{-1}$  with  $E/p=20 \text{ V/cm} \cdot \text{mm}$  Hg the density of radiant energy removed from a unit volume is higher, while the efficiency is slightly lower, than with  $E/p=10 \text{ V/cm} \cdot \text{mm}$  Hg. In addition, the use of high E/p facilitates the maintenance of a discharge of long duration owing to ionization of the gas in the electric field. Consequently, pumping with  $E/p=20 \text{ V/cm} \cdot \text{mm}$  Hg is preferable.

In the quasisteady mode of pumping, as seen from Fig. 2, a decrease in the relative concentration of electrons in the discharge entails an increase in the duration of the radiation pulse, a decrease in the density of the emitted energy, and a lowering of the efficiency starting with some n/p. The decrease in efficiency and  $Q_{\mathbf{r}}/p$  is due to relaxation losses of vibrational energy from the asymmetric mode of CO<sub>2</sub>. Consequently, an increase in the duration of the radiation pulse will be accompanied by losses of efficiency and of density of emitted energy.

The main characteristics of the shape of the radiation pulse are: the duration of the first generation peak, the radiant energy removed from a unit volume during this peak, and the duration of the radiation pulse in the mode established after the first peak. The dependence of  $Q_f/p$  on Q/p is presented in Fig. 3. Curves 1-4 pertain to the mixture  $1CO_2:1/4N_2:1.25He$ ; the respective durations of pumping equal  $pt_p = (0.13; 0.13; 1.3; 1.3) \cdot 10^{-6} \sec \cdot atm$ . Curves 5 and 6 pertain to the mixture  $1CO_2:2N_2:6\cdot 10^{-3}H_2O$ ; the values of  $pt_p$  are indicated on the graphs, as in Fig. 1.

It is seen from Fig. 3 that a low value of  $Q_f/p = (1-2) \cdot 10^{-3} \text{ J/cm}^3 \cdot \text{atm}$  is realized in a mixture with a high N<sub>2</sub> content (curves 5 and 6). The calculations show that in this case the duration of the first peak is ~10<sup>-6</sup> sec. In a mixture with a high content of CO<sub>2</sub> the value of  $Q_f/p$  grows approximately by a linear law with an increase in Q/p (see Fig. 3, curves 1-4). This nature of the dependence is due to the linear increase in energy in the asymmetric mode of CO<sub>2</sub> up to the time of the start of generation, since with a fixed pt<sub>p</sub> the increase in Q/p is achieved through an increase in  $Q_f/p$ , which is also explained by an increase in the pumping power. The high density  $Q_f/p=0.2 \text{ J/cm}^3 \cdot \text{atm}$  of the radiation in the first peak with  $E/p=10 \text{ V/cm} \cdot \text{mm}$  Hg is achieved owing to the high efficiency (~ 0.5 [4]) of the direct process of energy transfer from the electrons to the asymmetric mode of CO<sub>2</sub>. The duration of the first peak of emission in this case is equal to ~ 1.2 \cdot 10<sup>-7</sup> sec and is almost independent of Q/p. A similar nature of the dependence is noted in the experiments of [13]. The duration of generation grows with an increase in Q/p, but starting with Q/p=0.2 J/cm<sup>3</sup>  $\cdot \text{atm}$  it remains practically constant:  $pt_{T} = 7 \cdot 10^{-7} \sec \cdot \text{atm}$  with  $pt_{p} = 1.3 \cdot 10^{-7} \sec \cdot \text{atm}$  while  $pt_{T} = 1.4 \cdot 10^{-6} \sec \cdot \text{atm}$  with  $pt_{p} = 1.4 \cdot 10^{-6}$ 

In the formation of a radiation pulse of long duration it is important to know the radiation intensity in the established mode of generation.

The ratio of the maximum intensity in the established mode (after the first peak) to the product  $Lp^2$  is presented in Fig. 4 as a function of Q/p, as is the duration of generation  $pt_r$ . The composition of the active medium was  $1CO_2: 2N_2: 6 \cdot 10^{-3}H_2O$ , the solid lines correspond to E/p=10 V/cm·mm Hg, the dashed lines to E/p=20 V/cm·mm Hg, and the duration of pumping is indicated by numbers near the dots on the graphs, as in Fig. 1. The pumping durations were assigned in the calculations  $(pt_p=4\cdot 10^{-5} \text{ and } 6\cdot 10^{-5} \text{ sec} \cdot \text{atm})$  and the electron concentration in the discharge (and hence Q/p) was varied. When Q/p < 0.25 J/cm<sup>3</sup> · atm in the pumping modes under consideration  $(pt_p \ge 2 \cdot 10^{-5} \text{ sec} \cdot \text{atm})$  generation practically occurs only during the excitation of the active medium. Therefore, the increase in the duration of the radiation pulse takes place owing to shortening of the lag time of generation relative to the start of pumping. When Q/p  $\ge 0.25$  J/cm<sup>3</sup> · atm the discharge power is sufficiently great and the cutoff of generations is due to heating of the gas and populating of the lower laser level. Therefore, an increase in Q/p (through an increase in the discharge power) leads to a decrease in the duration of the radiation pulse. At the optimum value of Q/p=0.35 J/cm<sup>3</sup> · atm the radiation intensity, normalized to a unit length of active medium and to the square of the pressure, is equal to ~2 kW/cm<sup>3</sup> · atm<sup>2</sup>, while the product of the duration of the radiation pulse times the pressure is ~3 · 10<sup>-5</sup> sec · atm.

Thus, the investigation conducted made it possible to establish the optimum conditions for the formation of short and long radiation pulses. For the obtainment of pulses with a duration  $p_T = 3 \cdot 10^{-5}$  sec  $\cdot$  atm the optimum is the mixture  $1CO_2: 2N_2$  with an admixture of one of the gases in the following amount:  $p_{H_2O} = 2 \cdot 10^{-3}$  p,  $p_{He} = 0.4p$ , or  $p_{H_2} = 0.04p$ . The highest density of emitted energy is achieved with E/p = 20 V/cm·mm Hg. The optimum density of energy absorbed in the discharge is Q/p = 0.35 J/cm<sup>3</sup>  $\cdot$  atm and the efficiency is 0.18. The formation of short powerful radiation pulses with  $p_T = 10^{-7}$  sec  $\cdot$  atm and  $Q_f/p = 0.2$  J/cm<sup>3</sup>  $\cdot$  atm requires high pumping powers: Q/p = 0.5-1 J/cm<sup>3</sup>  $\cdot$  atm,  $p_T = 10^{-7}$  sec  $\cdot$  atm, E/p = 10 V/cm  $\cdot$  mm Hg. The optimum composition of the active medium is  $1CO_2: 1/4N_2$  with the addition of one of the gases in an amount  $p_{He} = p$ ,  $p_{H_2} = 0.1p$ , or  $p_{H_2O} = 2 \cdot 10^{-2}p$ . The data presented in the report are valid for pressures p = 0.1-5 atm, which facilitates the selection of the parameters for concrete  $CO_2$  lasers.

The author wishes to thank R. I. Soloukhin for support of the present work and A. G. Ponomarenko for assistance in the work.

## LITERATURE CITED

- 1. N. G. Basov, É. M. Belenov, V. A. Danilychev, and A. F. Suchkov, "Electrical ionization lasers based on compressed carbon dioxide," Usp. Fiz. Nauk, 114, No. 2, 213 (1974).
- S. P. Bugaev, Yu. I. Bychkov, E. K. Karlova, N. V. Karlov, B. N. Koval'chuk, G. P. Kuz'min, Yu. A. Kurbatov, G. A. Mesyats, V. M. Orlovskii, A. M. Prokhorov, and A. M. Rybalov, "A pulsed CO<sub>2</sub> laser with an energy of 500 J," Zh. Tekh. Fiz., Pis'ma Red., <u>1</u>, No. 10, 492 (1975).
- 3. W. L. Nighan, "Electron energy distributions and collision rates in electrically excited N<sub>2</sub>, CO<sub>2</sub>, and CO," Phys. Rev., 2a, No. 5, 1989 (1970).
- 4. A. N. Lobanov and A. F. Suchkov, "Distribution function and energy balance of electrons in a carbon dioxide electrical ionization laser," Kvant. Élektron., 1, No. 7, 1527 (1974).
- 5. A. S. Biryukov and V. F. Gordiets, "Kinetic equations of relaxation of vibrational energy in a mixture of multiatomic gases," Zh. Prikl. Mekh. Tekh. Fiz., No. 6, 29 (1972).

- 6. L. V. Dubovoi, V. A. Zaitsev, V. N. Poponin, and V. F. Shanskii, "Choice of optimum pumping power for a pulsed CO<sub>2</sub> laser with a non-self-maintained discharge," Kvant. Élektron., 2, No. 7, 1415 (1975).
- 7. G. A. Vedernikov, V. N. Karnyushin, and R. I. Soloukhin, "Quasisteady mode of generation of a CO<sub>2</sub> laser excited by a non-self-maintained electrical discharge," Zh. Prikl. Mekh. Tekh. Fiz., No. 2, 3 (1975).
- 8. L. V. Dubovoi, V. A. Zaitsev, and V. P. Poponin, "Effect of an admixture of hydrogen on efficiency of generation of a pulsed CO<sub>2</sub> laser excited by a non-self-maintained discharge," Zh. Tekh. Fiz., Pis'ma Red., 1, No. 9, 411 (1975).
- 9. N. G. Basov, I. A. Berezhnoi, V. A. Boiko, V. A. Danilychev, V. D. Zvorykin, V. V. Ignat'ev, I. V. Kholin, and A. Yu. Chugunov, "One possibility of using electrical ionization CO<sub>2</sub> lasers for purposes of laser thermonuclear synthesis," Zh. Tekh. Fiz., Pis'ma Red., 1, No. 24, 1105 (1975).
- A. G. Ponomarenko, R. I. Soloukhin, and V. N. Tishchenko, "Optimization and limiting energetic characteristics of CO<sub>2</sub> lasers," Zh. Prikl. Mekh. Tekh. Fiz., No. 5, 120 (1975).
- 11. A. S. Biryukov, V. K. Konyukov, A. I. Lukovnikov, and R. I. Serikov, "Relaxation of vibrational energy of the (00<sup>0</sup>1) level of the CO<sub>2</sub> molecule," Zh. Éksp. Teor. Fiz., <u>66</u>, No. 4, 1248 (1974).
- V. V. Danilov, É. P. Kruglyakov, and E. V. Shun'ko, "Measurement of probability of the P(20) (00<sup>0</sup>1-10<sup>0</sup>0) transition of CO<sub>2</sub> and impact broadening in collisions of CO<sub>2</sub>, N<sub>2</sub>, and He," Zh. Prikl. Mekh. Tekh. Fiz., No. 6, 24 (1972).
- Yu. V. Afonin, "A compact source of accelerated electrons and an electrical ionization CO<sub>2</sub> laser for space research," Candidate's Dissertation, Institute of Theoretical and Applied Mechanics Siberian Branch, Academy of Sciences of the USSR, Novosibirsk (1975).

## INFLUENCE OF SUPERSONIC FLOW HEATING ON THE GAIN OF A CARBON DIOXIDE GASDYNAMIC LASER

S. A. Losev and V. N. Makarov

UDC 621.375.826

The statement of the problem in the present article is the same as in our preceding work [1], namely optimization by the method of configurations with respect to initial conditions, composition, and nozzle geometry. The transition to dimensionless variables shows that the optimization parameters in the given problem are  $p_0$ ,  $T_0$ ,  $\xi_i$ ,  $\lambda$ , and  $\beta_j$ , where  $p_0$  and  $T_0$  are the initial pressure and initial temperature;  $\xi_i$  denotes the mole fractions;  $\lambda = p_0 l$  (l is a characteristic length); and  $\beta_j$  denotes parameters determining the dimensionless function A/A<sub>\*</sub>, where A and A<sub>\*</sub> are the nozzle cross sections at an arbitrary point and at the geometrical critical point, respectively. In [1] we adopted as  $\beta_j$  the values of the derivatives at certain fixed mesh-points  $\bar{x}_j = x_j/L$ , where  $\bar{x}$  is the dimensionless distance along the x axis, referred to the nozzle length L. Quadratic approximation was applied to determine A/A<sub>\*</sub> between the mesh-points. We take l = L, so that for plane-parallel flows  $\beta_j = \alpha_j L = \frac{2 t g \theta_j}{h_*} L$ , where  $\theta_j$  denotes the slope angles of the nozzle contour at the points  $x_j = \bar{x}_j L$ ; and  $h_*$  is the height at the critical cross section of the nozzle. We consider fixed values of the initial pressure  $p_0$ , so that  $T_0$ ,  $\xi_i$ ,  $\alpha_j$ , and L can be taken as the optimization parameters [1]. Unlike [1], the parameters  $\alpha_j$  (j = 1, 2) can assume negative values.

The results of optimization with respect to the indicated parameters as a function of the initial pressure  $p_0$  are given in Fig. 1 for a  $CO_2 + N_2 + He$  mixture. These results show that following the large expansion of the supersonic flow in the geometrical critical zone of the nozzle a certain downstream constriction is observed in the optimal regime, i.e.,  $\alpha_1$  and  $\alpha_2$  can assume negative values. For small initial pressures  $(p_0 < \bar{p}_0)$  we have  $\alpha_1 > 0$  and  $\alpha_2 < 0$ , i.e., flow constriction must occur closer to the nozzle exit, and for large initial pressures  $(p_0 > \bar{p}_0)$  we have  $\alpha_1 < 0$  and  $\alpha_2 > 0$ , i.e., constriction is observed in the middle part of the supersonic flow region of the nozzle. For pressures  $p_0 \approx \bar{p}_0$  we find that  $\alpha_1 \sim 0$  and  $\alpha_2 \sim 0$ , i.e., after the initial expansion and parabolic transition ( $\alpha_0 > 0$ ) the flow cross section remains practically invariant up to the optimum distance L. For the

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 15-18, May-June, 1977. Original article submitted May 17, 1976.

This material is protected by copyright registered in the name of Plenum Publishing Corporation, 227 West 17th Street, New York, N.Y. 10011. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for \$7.50.